Structural damage assessment with antiresonances versus mode shapes using parallel genetic algorithms

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SUMMARY

Antiresonances have become an attractive alternative in structural damage assessment. They can be identified easier and more accurately than mode shapes, and still providing the same information. Antiresonances are derived from point frequency response functions (FRFs) or from transfer FRFs. However, antiresonances from transfer FRFs are very sensitive to small structural changes, and the matching between numerical and experimental antiresonances is affected. This problem is solved if antiresonances from point FRFs are used. However, it implies an experimental procedure that differs from a common modal testing, which may become not practical or too expensive. This paper proposes a damage detection method able to deal with transfer antiresonances. The inverse problem is handled by a Parallel Genetic Algorithm. In this case, a perfect match between the antiresonances is not required because the optimization is not gradient based. Moreover, the matching can change at each step and the optimization is not affected. The algorithm is verified with two experimental cases: an exhaust system of a car with a single fatigue crack and a tridimensional space frame structure with single and multiple damage scenarios. Results are compared with the ones obtained using mode shapes. Damage detected is consistent with the experimental damage in both cases.

KEY WORDS: damage assessment; antiresonances; parallel genetic algorithm; damage penalization

1. INTRODUCTION

Traditional damage assessment methods make use of modal information as natural frequencies and mode shapes. In general, natural frequencies detect and quantify the presence of damage, while mode shapes located it. Natural frequencies can be accurately identified, but this is not the case for mode shapes. Mode shapes are usually accurate to within 10% at best; this can cause problems in the detection of the exact damage location. The use of the Frequency Response Function (FRF) is also an alternative [1,2]. The advantage is that no modal extraction is necessary, thus contamination of the data with modal extraction errors is avoided. However, complex FRF data with noise can make the convergence process very slow and often numerically unstable as was found by Imregun et al. [2]. Furthermore, the success of the method is highly dependent on the selection of the frequency points. Lammens [3] addresses how a poor selection of the frequency points can lead to an unstable updating process and inaccurate results. FRFs have also the disadvantage that they cannot be identified from output only modal...
analysis with ambient excitation, thus the excitation by an artificial force is always required. These limitations have motivated the study of new damage indicators extracted from FRFs.

Recently, great attention has been given to the possible use of antiresonances in structural damage detection. Antiresonances are an attractive alternative because they can be determined easier and with less error than mode shapes. Wahl et al. [4] study the possibilities of antiresonances as indicators of structural modifications. They conclude that antiresonances may lead to future applications of identification and location of structural faults, although no implementation is attempted. Mottershead [5] shows that antiresonances sensitivities are linear combination of eigenvalues and mode shapes sensitivities. Hence, antiresonances are an alternative to mode shapes since they provide the same information. As natural frequencies, antiresonances are located along the frequency axis and can be estimated from experimental FRFs more accurately than mode shapes. In addition, antiresonances can, in principle, be identified from operational modes [6]. Antiresonances are also very sensitive to small structural changes, which makes them good damage indicators. Despite these advantages, the use of antiresonances is still under development and the application of antiresonances to structural damage detection has not been fully investigated.

Antiresonances can be derived from point FRFs, where the response coordinate is the same as the excitation coordinate; or from transfer FRFs, where the response coordinate differs from the excitation coordinate. Point FRFs are preferred because matching problems arise when antiresonances from transfer FRFs are used. Moreover, the distribution of the transfer antiresonances can be significantly modified with small structural changes [7]. On the other hand, the procedure to obtain point FRFs differs from common modal testing, i.e. the excitation degree of freedom (DOF) is moved together with the response DOF. This may become not practical or too expensive.

D’Ambrogio and Fregolent [7] use antiresonances and natural frequencies to update the model of a frame structure. With antiresonances from point FRFs the method is robust and leads to good results. On the contrary, with transfer antiresonances the method is very unstable. Only with a careful selection of the updating parameter and a good match between experimental and numerical antiresonances they could reach results. Transfer antiresonances are used by Jones and Turcotte [8] to update a six-meter flexible truss structure. The correctness of the updated model is studied by using it to detect the damage. The GARTEUR structure is updated with an antiresonance-based method by D’Ambrogio and Fregolent [9]. The unmeasured point FRFs are synthesized through a truncated modal expansion. In a later work [10], they propose the use of zeros from a truncated expansion of the identified modes; they refer to these zeros as ‘virtual antiresonances’. Results are compared with an updating method using MAC and natural frequencies. The updated models using either true or virtual antiresonances were more accurate than with MAC. Dilema and Morassi [11] study the problem of crack detection in beams using natural frequencies and antiresonances. They find that the use of antiresonances help to avoid non-uniqueness of the damage location that occurs when only natural frequencies are used. However, they also find that experimental noise and modeling errors are usually amplified when antiresonances are included. Bamions et al. [12] propose a scheme for crack location on beams. They use the shift in the first antiresonance versus the measuring position to detect and locate a crack. The changes in the antiresonances and natural frequencies are used by Inada et al. [13] to locate and quantify delamination of a composite beam. Wang and Zhu [14] show that antiresonances are local parameters, which provide important information for damage assessment. They propose a method to identify cracks in beams, which makes use of natural frequencies and antiresonances from point FRFs. Nam et al. [15] study the improvement in the performance of a parameter estimation algorithm by adding additional spectral information. The basic spectral information originates from the natural frequencies and the additional information from the antiresonances and static compliance dominant frequencies. Antiresonances are obtained from point and transfer FRFs. The method is evaluated with a numerical spring mass system. They conclude that the accuracy of the algorithm can be improved with the use of additional spectral information as antiresonances and compliance dominant frequencies.

This work intends to use transfer antiresonances and natural frequencies to solve the inverse problem of model-based damage assessment. Here the optimization is particularly challenging.
and a robust optimization algorithm is needed. Parallel Genetic Algorithms (PGA) is shown to be a robust and reliable optimization algorithm in structural damage assessment [16]. The main problem with transfer antiresonances is matching the numerical and the experimental antiresonances. With Genetic Algorithms a perfect matching is not required because the search is not gradient based. Moreover, the matching between experimental and numerical antiresonances can change at each step and the optimization is not affected. The Genetic Algorithm is a global searching process based on Darwin's principle of natural selection and evolution. The crossover is considered its main search operator. As a consequence, many types of crossover have been developed. Some of them are as follows: single-point crossover, two-point crossover [17], uniform crossover [18], flat crossover [19], arithmetic crossover [20], heuristic crossover [21], blend crossover [22], BLX-α crossover [22], and many more. Each crossover technique directs the search in different areas near the parents, some of them use more exploration (or interpolation) and others more exploitation (or extrapolation). For the algorithm to be successful there must be an adequate balance between exploration and exploitation [23]. Herrera et al. [24] show that by combining different types of crossovers the effectiveness of the search is improved.

GAs are inherently slow when they work with complicated or time-consuming objective functions. There are several methods to increase the speed on these cases. One is to ensure that identical chromosomes are not evaluated twice; it is then necessary to search in the population for identical twins. This is only worth the effort when the cost function evaluation takes longer than the search. An alternative is to work with a hybrid-GA; it combines the power of the GA with the speed of a local optimizer. Thus, the GA finds the regions of the optimum, and then the local optimizer takes over to find the minimum [25]. PGAs are also an attractive alternative; they are particularly easy to implement and provide a superior numerical performance. PGAs in addition to being faster than sequential GAs lead to better results. According to Punch’s [26] research, as the number of parallel populations increases, there is a super linear speedup in solution time. This indicates that there is not only a speedup from multiple processors, but also an increase in performance because of multiple populations. Multiple populations allow specification; each population searches in a different area of the solution space, performing a better search than sequential GAs. Therefore, many authors run PGAs in a single machine and still obtain better results than with sequential GAs [27]. Meruane and Heylen [16, 25] investigated the advantages of PGAs in a structural damage detection problem using operational data. Two experimental structures are used to verify the approach: a test structure of an airplane and a multiple cracked reinforced concrete beam. The performance of the parallel algorithm was compared to one of the sequential cases. It was concluded that PGA always provides an improved and faster search in the solution space compared to sequential GAs.

A model-based algorithm using transfer antiresonances and natural frequencies is implemented to detect structural damage. The performance of the approach using antiresonances is compared with the approach using mode shapes. Thus, the capability of antiresonances as indicators of structural modifications is compared to mode shapes. The optimization is handled by a PGA. To ensure a balanced search, each population uses a different crossover and two mutation operators. The damage penalization proposed by Meruane and Heylen [16, 25] is used to successfully avoid false damage detection. An automated selection of the penalization parameter is implemented. The damage assessment algorithm is applied to two experimental cases. First, an exhaust system of a car with a single fatigue crack is investigated. The crack is introduced by a fatigue test; three increasing levels of damage are studied. A tridimensional space frame structure with single and multiple damage scenarios serves as a second test case. Damage detected is consistent with the experimental damage in both cases.

2. FORMULATION OF THE OPTIMIZATION PROBLEM

Damage is represented by an elemental stiffness reduction factor \( \beta \), defined as the ratio between the stiffness reduction to the initial stiffness. This is the simplest method to model damage. Although this model has problems in matching damage severity to crack depth and is affected
by mesh density, Friswell and Penny [28] demonstrate that at low frequencies this method can
correctly model a crack. It is shown that a more detailed model does not substantially improve
the results from damage assessment. The stiffness matrix of the damaged structure \([K_d]\) is
expressed as a sum of element matrices multiplied by reduction factors,

\[
[K_d] = \sum_i (1 - \beta_i)[K_i]
\]  

The value \(\beta_i = 0\) indicates that the element is undamaged whereas \(0 < \beta_i \leq 1\) implies partial or
complete damage.

The problem of detecting damage is a constrained nonlinear optimization problem, where the
damage reduction factors \(\beta_i\) are defined as updating parameters. Two objective functions are
employed. The first correlates transfer antiresonances and natural frequencies. The second
correlates mode shapes and natural frequencies. To avoid the need of an accurate numerical
model, its initial correlation is added to the objective function. The inclusion of the initial
modeling errors in the objective function has been used by Asce and Xia [29], Titurus et al [30]
and Meruane and Heylen [16].

The error in natural frequencies is represented by the ratio between the experimental and
analytical eigenvalues,

\[
\varepsilon_{\lambda,i}(\beta) = \frac{\lambda_{A,i}(\beta)}{\lambda_{E,i}} - \frac{\lambda_{A,0,i}}{\lambda_{E,0,i}}
\]  

The superscripts \(A\) and \(E\) refer to analytical and experimental respectively and the
subscript 0 refers to the initial undamaged state. \(\lambda_i\) is the \(i\)th eigenvalue and \(\omega_i\) is the \(i\)th
natural frequency.

The error in antiresonances is represented by the ratio between the experimental and
analytical antiresonances

\[
\varepsilon_{\text{or},i,n}(\beta) = \frac{\omega_{A,i,n}(\beta)}{\omega_{E,i,n}} - \frac{\omega_{A,0,i,n}}{\omega_{E,0,i,n}}
\]  

The superscripts \(A\) and \(E\) refer to analytical and experimental and the superscript 0 refers to
the initial undamaged state. \(\omega_{r,i,n}\) is the \(i\)th antiresonance of the \(n\)th FRF.

The difference between modes is represented by the Modal Assurance Criterion (MAC). MAC is defined by Allemang and Brown [31] as,

\[
\text{MAC}_i = \frac{(\phi_{A,i}^T \phi_{E,i})^2}{(\phi_{A,i}^T \phi_{A,i})(\phi_{E,i}^T \phi_{E,i})}
\]  

\(\phi_i\) is the \(i\)th mode shape. MAC is a factor that expresses the correlation between two modes.
A value of 0 shows no correlation, whereas a value of 1 shows two completely correlated modes.

The error is defined by

\[
\varepsilon_{\text{MAC},i}(\beta) = (\text{MAC}_i(\beta) - \text{MAC}_{0,i})^2
\]  

The subscript 0 refers to the initial correlation of the undamaged modes. If the number of
measured DOFs is lower than the numerical DOFs, a partial MAC is used. Hence, no mode
shape expansion is needed.

In Equations (2), (3) and (5), the goal is not to reach a perfect match between the
numerical and experimental parameters, but rather to reach the same correlation than in the
undamaged case. This considers initial errors in the numerical model. If a damaged mode or
antiresonance cannot be matched with an undamaged one, its initial correlation is set equal to 1.
This occurs in cases where after the introduction of damage, new modes or antiresonances
are detected.

Two objective functions are considered. The first correlates natural frequencies and
antiresonances. The second correlates mode shapes and natural frequencies. The objective
functions are the normalized sum of the errors plus a damage penalization term

\[ J_1(\{\beta\}) = \frac{F_l(\{\beta\})}{F_{l,0}} + \frac{F_{\text{or}}(\{\beta\})}{F_{\text{or},0}} + F_D(\{\beta\}) \]

\[ J_2(\{\beta\}) = \frac{F_l(\{\beta\})}{F_{l,0}} + \frac{F_{\text{MAC}}(\{\beta\})}{F_{\text{MAC},0}} + F_D(\{\beta\}) \]

\[ F_l(\{\beta\}) = \sum_j \|e_{l,j}(\{\beta\})\| \]

\[ F_{\text{or}}(\{\beta\}) = \sum_n \sum_j \|e_{\text{or},j}(\{\beta\})\| \]

\[ F_{\text{MAC}}(\{\beta\}) = \sum_j \|e_{\text{MAC},j}(\{\beta\})\| \]

\( F_{l,0}, F_{\text{or},0}, F_{\text{MAC},0} \) refers to the initial values of the sums (\( \beta = 0 \)).

\( F_D \) is a damage penalization function. Damage penalization helps to avoid false damage detection because of experimental noise or numerical errors [16,25]. Two damage penalization functions are used:

\[ F_{D,1} = \gamma_1 \sum_i \beta_i \]

\[ F_{D,2} = \gamma_2 \sum_j \delta_i, \quad \delta_i = \begin{cases} 1 & \beta_i > 0 \\ 0 & \beta_i = 0 \end{cases} \]

The first penalizes the total amount of damage. The second, on the other hand, penalizes the number of damage locations. Depending on the damage pattern expected, one can use the first function, the second or a combination of both. The value of the constants \( \gamma_1 \) and \( \gamma_2 \) depend on the confidence in the numerical model and the experimental data.

The optimization problem is defined as

\[ \min J(\{\beta\}) \]

subject to \( 0 \leq \beta_i \leq 1 \)

(8)

3. IDENTIFICATION AND MATCHING OF ANTIRESONANCES

3.1. Experimental antiresonances

Here antiresonances are identified from experimental FRFs by ‘dip-picking’ [7]. In this technique antiresonances are selected by picking the dips from the magnitude plot of a given FRF that have an associate change in \(+180^\circ\) in the phase plot. This technique is similar to the ‘peak picking’ technique for resonances. When using this technique, the errors in the estimated antiresonances are given mainly by the frequency resolution of the FRFs. If the dips in the FRFs are difficult to determine, antiresonances can be identified by a curve-fitting technique based on the rational fraction polynomial representation of FRFs [32].

Figure 1 shows two examples of the antiresonance identification followed. The left graphs show the identification for one FRF from the exhaust system, the right graphs for one FRF from the space frame structure.

3.2. Numerical antiresonances

Antiresonances are the zeros of the FRFs. The zeros of the \( k, j \)th FRF are given by the eigenvalues of \((M^{-1}K)_{k,j}\), where \( K \) and \( M \) represent the stiffness and mass matrices. The subscripts \( k, j \) denote that the \( k \)th row and \( j \)th column have been deleted [5]. If \( k \neq j \), the matrix is not symmetrical, hence some of the eigenvalues may be negative or complex; these values must not be considered as antiresonances.
3.3. Matching experimental and numerical antiresonances

Matching numerical and experimental antiresonances becomes a difficult task, mainly because the antiresonances distribution is significantly altered with small structural changes. Because the optimization algorithm used here is not gradient based, a perfect matching is not necessary. Moreover, the matching can change at each step and the optimization is not affected. Having this in consideration, the numerical and experimental antiresonances are paired each time the objective function is evaluated. Each experimental antiresonance is paired with the closest numerical antiresonance. Because the number of experimental and numerical antiresonances may not be the same, a numerical antiresonance is allowed to be paired with one or more experimental antiresonances.

4. DAMAGE DETECTION ALGORITHM

Meruane and Heylen [25] investigate the application of a sequential GA for structural damage detection. The performance of five fundamental functions based on modal data are studied. As a result, the objective function that combines the frequency difference errors and MAC values is selected. The set-up of the GA operators and parameters is addressed, providing guidelines to their selection in similar damage detection problems. In addition, it is proposed to use a damage penalization that satisfactorily avoids false damage detection due to experimental noise or numerical errors. Later in [16] the algorithm is extended to PGAs and the following improvements are introduced: the initial errors in the numerical model are included in the objective function, the objective function is based on operational modal data and a second form of damage penalization is suggested. Results obtained in References [16] and [25] are promising: a fast and efficient algorithm able to detect experimental damage accurately has been developed. Nevertheless, many parameters and operators must be chosen beforehand: different applications require different combinations of parameters and operators, which makes the application of the algorithm sometimes difficult. Here, the results obtained in [16] and [25] are used to develop a damage detection algorithm that can be used in any kind of structure with no need of selecting parameters or operators.

The optimization is handled by a parallel GA programmed in Matlab and run in a cluster. The gene of each chromosome is the stiffness reduction factor of each element. Each chromosome represents one possible damage distribution. The algorithm employs a multiple population GA with five populations and a neighborhood migration. The penalization function selected is the sum of $F_{D_1}$ and $F_{D_2}$ (see Equation (7)) with $\gamma_1 = \gamma_2 = \gamma$.

A normalized geometric selection is used as was recommended by Meruane and Heylen [25]. To ensure an effective search with an adequate balance between exploration and exploitation, each population works with a different crossover, being the following ones: arithmetic crossover
[20], heuristic crossover [21], BLX-0.5 crossover [22], two-point crossover [17], and uniform crossover [18]. In addition, each population applies both boundary and uniform mutations. Each population has a size of 40 individuals and the crossover and mutation probabilities are as follows: $p_c = 0.80$ and $p_m = 0.02$ respectively.

The migration interval is automatically adjusted. If a population has no improvement after a predefined number of generations, the GA stops and exchanges the individuals with their neighbors. This exchange of individuals is synchronous, i.e. the algorithm waits until the five populations are ready to perform the migration. At each migration, each population sends its best individual, whereas its worse individual is replaced by the received individual. Before each migration, the best individuals from all populations are compared, if they are all the same the optimization is finished. Figure 2 illustrates this process.

Because the appropriate value for $\gamma$ is not known, its value is dynamically adjusted as shown in Figure 3. First the solution with $\gamma = 0$ is computed, next the value of $\gamma$ is increased by $\delta$ and the solution is recomputed. This process is iterated until a stable solution is reached and stops. The solution is defined stable if after three consecutive steps it remains the same. The value of $\delta$ used is 0.02.

![Figure 2. Parallel optimization.](image)

![Figure 3. Damage detection algorithm.](image)
5. APPLICATION CASES

5.1. Car exhaust system

The structure is a car exhaust system as shown in Figure 4. The dimensions are as follows: length, 2.3 m; width, 0.45 m. The exhaust pipe has a diameter of 38 mm. The structure is suspended by soft springs and it is excited randomly by an electrodynamic shaker. The response is captured by 20 accelerometers. The test is performed in a frequency range of 0–1024 Hz with a frequency resolution of 0.25 Hz.

The numerical model shown in Figure 5 is built in Matlab with 2D beam element and concentrated inertias for the masses. The model has 47 beam and 3 inertia elements, with 144 DOF. Table I presents the correlation of the numerical model after being updated with the experimental data. The minimum MAC value is 0.98 and the maximum frequency difference is 2.78%.

A single fatigue crack with three increasing levels of damage is introduced to the structure. To develop the fatigue crack, the structure is clamped over a 6-DOF shaker table as shown in Figure 6. The shaker excites the structure in the vertical direction with frequencies from 0 to 50 Hz. The first two modes of the clamped structure are excited, therefore inducing large displacement vibrations. The largest strain is located at the welded connections between the resonator and exhaust pipe. Visual observations and strain gauge readings are used during the test to detect the presence of damage. The fatigue test continues until a visible macro-crack is

![Figure 4. Experimental set-up.](image1)

![Figure 5. Finite element model and element numbering.](image2)

<table>
<thead>
<tr>
<th>Mode</th>
<th>MAC</th>
<th>(\omega_E)</th>
<th>(\omega_A)</th>
<th>(\Delta\omega) (%)</th>
</tr>
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observed. Figure 7(a) shows the crack, which is located in the welded connection between elements 30 and 31 (see Figure 5) and covers around 60% of the pipe perimeter. The fatigue test is done again twice to grow the crack. Figure 7(b) shows the second damage level; here the structure has already failed because of unstable crack propagation. The open crack covers around 70% of the perimeter. The last damage level is shown in Figure 7(c), and the crack covers around 85% of the perimeter.

After each crack size, the structure is subjected to an experimental modal analysis. Figure 4 shows the experimental setup. Table II summarizes the observed changes in the modal properties at each case. The change in natural frequencies is calculated with respect to the undamaged frequencies. MAC values are calculated between the undamaged and damaged modes. There is a significant frequency reduction in the three cases, showing the presence of damage. In the first case, no significant reduction in the MAC values can be observed. The second and third cases cause a drop in the MAC values for modes 1 and 2.

Damage is detected first with the objective function $J_1$ and then with $J_2$ from Equation (6) through the algorithm described in Section 4. The 47 beam elements are considered as possible damage locations. The first four mode shapes and natural frequencies are used. All the identified antiresonances in the range 0–300 Hz from the 20 measured FRFs are used. The number of identified antiresonances varies from 26 to 34 depending on the case.

Figure 8 show the results. The left graphs present the results with antiresonances and the right graphs with mode shapes. If $\gamma$ equals zero, a significant amount of false damage is detected. This false damage is sequentially reduced by increasing the value of $\gamma$ until a stable solution is reached. Once the stable solution is reached the algorithm stops. Table III shows the results
obtained on each case. The experimental crack is located between elements 30 and 31, thus we expect to detect damage in one of these two elements. In the first case, both algorithms are successful in detecting the damage, although with antiresonances false damage is still detected. In the second and third cases, the approach based upon antiresonances detects damage in element 32 and there is no difference in the damage detected for cases 2 and 3. On the other hand, the approach based upon modes locates the damage correctly in element 31. In addition, the method is able to detect the difference between cases 2 and 3. In case 3, damage is detected also in element 32, which indicates a larger damage than in case 2.

5.2. Space frame structure

Figure 9 shows the experimental setup, the structure is a tridimensional statically indeterminate space truss, consisting of 43 members and 20 joins. The same structure was used by Meruane and

<table>
<thead>
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<th>Case</th>
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<tr>
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<td>32</td>
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Heylen [25]. The dimensions of the structure are as follows: length, 3 m; width, 0.5 m; height, 0.5 m. Each bar has a diameter of 20 mm and the material is aluminium. The structure is suspended by soft springs to simulate a ‘free–free’ boundary condition. The response is measured by 26 accelerometers. The frequency range of analysis covers the range from 0 to 256 Hz with a resolution of 0.25 Hz. This range contains all global modes and also a list of local modes.

The Finite Element Model is built on Matlab with 3D beam elements for the bars and concentrated inertia elements for the joins. Each bar is modeled with four beam elements as shown in Figure 10. Rotational DOF were deleted from the FE model using the Guyan reduction technique [33]. As a result, the model has 447 DOF and 192 elements. Twelve global modes are computed with frequencies between 12 and 125 Hz, subsequent modes are local bar modes. Damage is considered at the bar level, so all elements on each bar are grouped in a single macro element.

Table IV presents the correlation between the undamaged numerical and experimental modes. The maximum frequency difference is 2.3% and the minimum MAC value is 0.925. Four damage cases are studied. In the first three cases, a few aluminum bars are replaced by plastic bars, thus the stiffness and mass of the replaced bar is reduced. Stiffness reduction is 90% and mass reduction is 67%. The replaced bars on each case are as follows: case 1, bar 6; case 2, bars 6 and 25; case 3, bars 6, 25 and 43. In case 4 bar 34 was completely removed from the structure. Figure 11 shows the bar numbering.

Table V shows the changes in the modal properties after the introduction of damage. The change in natural frequencies is calculated with respect to the undamaged frequencies. MAC values are calculated between the undamaged and damaged modes. In cases 1, 2 and 3, there is a general reduction in frequencies. The frequency of mode 12 is significantly reduced in cases 2 and 3. Significant drops in the MAC values are observed in modes 10 and 11. In the fourth case, some frequencies are reduced while others are increased. A significant drop in the MAC values of modes 11 and 12 is noticed. Although it is not shown in Table V, in cases 1, 2 and 4 a new mode is detected with a frequency of 139, 135 and 46.7 Hz respectively, and two new modes are detected in case 3 with frequencies of 135 and 143 Hz.
To model correctly the experimental damage, i.e. replacing or removing a bar, a mass reduction factor must also be considered. For convenience, the mass reduction factor is defined proportional to the stiffness reduction factor, and this allows to update only the stiffness parameters as in a general damage detection algorithm.

Damage is detected first with $J_1$ and later with $J_2$ from Equation (6), through the algorithm described in Section 4. The 43 bars are considered as possible damage locations. All global mode shapes and natural frequencies are used. This includes the 12 modes listed in Table IV and new

<table>
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<tr>
<td>11</td>
<td>0.986</td>
<td>58.87</td>
<td>58.80</td>
<td>0.13</td>
</tr>
<tr>
<td>12</td>
<td>0.980</td>
<td>115.56</td>
<td>115.76</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table V. Changes in experimental modes and frequencies after the introduction of damage.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\Delta\omega_E$ %</th>
<th>MAC</th>
<th>$\Delta\omega_A$ %</th>
<th>MAC</th>
<th>$\Delta\omega_E$ %</th>
<th>MAC</th>
<th>$\Delta\omega_A$ %</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.51</td>
<td>0.97</td>
<td>-4.82</td>
<td>0.96</td>
<td>-3.63</td>
<td>0.92</td>
<td>0.28</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>-3.81</td>
<td>0.99</td>
<td>-2.37</td>
<td>0.99</td>
<td>-1.32</td>
<td>0.88</td>
<td>-8.73</td>
<td>0.91</td>
</tr>
<tr>
<td>3</td>
<td>-2.44</td>
<td>0.99</td>
<td>-2.80</td>
<td>0.97</td>
<td>-3.10</td>
<td>0.93</td>
<td>2.12</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>-1.76</td>
<td>0.98</td>
<td>-1.65</td>
<td>0.96</td>
<td>-3.31</td>
<td>0.99</td>
<td>3.32</td>
<td>0.88</td>
</tr>
<tr>
<td>5</td>
<td>-5.69</td>
<td>0.91</td>
<td>-5.05</td>
<td>0.92</td>
<td>-4.67</td>
<td>0.92</td>
<td>1.02</td>
<td>0.81</td>
</tr>
<tr>
<td>6</td>
<td>-2.76</td>
<td>0.94</td>
<td>-3.24</td>
<td>0.92</td>
<td>-2.33</td>
<td>0.87</td>
<td>-8.86</td>
<td>0.98</td>
</tr>
<tr>
<td>7</td>
<td>-3.53</td>
<td>0.91</td>
<td>-3.16</td>
<td>0.88</td>
<td>-2.33</td>
<td>0.93</td>
<td>1.69</td>
<td>0.89</td>
</tr>
<tr>
<td>8</td>
<td>-0.64</td>
<td>0.97</td>
<td>-2.08</td>
<td>0.95</td>
<td>-2.73</td>
<td>0.80</td>
<td>-2.63</td>
<td>0.94</td>
</tr>
<tr>
<td>9</td>
<td>-0.98</td>
<td>0.97</td>
<td>-1.70</td>
<td>0.95</td>
<td>-3.11</td>
<td>0.98</td>
<td>-0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>-3.41</td>
<td>0.66</td>
<td>-3.63</td>
<td>0.65</td>
<td>-2.61</td>
<td>0.69</td>
<td>2.85</td>
<td>0.81</td>
</tr>
<tr>
<td>11</td>
<td>-3.37</td>
<td>0.61</td>
<td>-4.41</td>
<td>0.63</td>
<td>-2.48</td>
<td>0.61</td>
<td>3.10</td>
<td>0.43</td>
</tr>
<tr>
<td>12</td>
<td>-0.70</td>
<td>0.95</td>
<td>-35.20</td>
<td>0.88</td>
<td>-39.25</td>
<td>0.81</td>
<td>4.84</td>
<td>0.59</td>
</tr>
</tbody>
</table>

To model correctly the experimental damage, i.e. replacing or removing a bar, a mass reduction factor must also be considered. For convenience, the mass reduction factor is defined proportional to the stiffness reduction factor, and this allows to update only the stiffness parameters as in a general damage detection algorithm.

Damage is detected first with $J_1$ and later with $J_2$ from Equation (6), through the algorithm described in Section 4. The 43 bars are considered as possible damage locations. All global mode shapes and natural frequencies are used. This includes the 12 modes listed in Table IV and new
modes detected after the introduction of damage. All the identified antiresonances in the range 0–150 Hz from the 20 measured FRFs are used. The number of identified antiresonances varies from 53 to 95 depending on the case.

Figure 12 presents the results, with the left graph showing the results obtained with antiresonances and the right graph showing mode shapes. False damage is sequentially reduced by increasing the value of $\gamma$ until it is completely avoided. Table VI resumes the damage detected on each case. In cases 1, 2 and 3, the stiffness is reduced by 90%, whereas in case 4 by 100%.
In cases 1, 2 and 4, both algorithms are successful in detecting the damage, although in case 2 the method with mode shapes is more precise in quantifying the damage. In case 3, damage is successfully detected using modes shapes, but with antiresonances the method fails in detecting the 25th element as damaged.

6. CONCLUSIONS

A damage detection method using transfer antiresonances and PGAs has been implemented. False damage detection is avoided by damage penalization. The algorithm makes an automated selection of the penalization parameter. The objective function includes the initial errors in the numerical model; hence, it is not necessary to start from a very accurate numerical model. The damage assessment algorithm is applied to two experimental cases. First, an exhaust system of a car with a single fatigue crack. Second, a tridimensional space frame structure with single and multiple damage scenarios. Results are compared with the ones obtained using mode shapes.

In both application cases, the damage detected has a good correspondence with the experimental damage. The algorithm was successful on using antiresonances from transfer FRFs to detect structural damage. Hence PGAs are a good solution to handle this difficult optimization problem. Although, antiresonances are determined more accurately than mode shapes, this is not reflected on the results. Moreover, in the studied cases results were more precise when mode shapes were used. Further studies in the identification, selection and matching of the antiresonances should be conducted to improve the damage assessment results.

REFERENCES